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Review
APPLICATIONS OF MATHEMATICAL MODELING AND AGRICULTURAL CROP SIMULATION IN CUBA

Revisión bibliográfica
Aplicaciones de la modelación matemática y la simulación de cultivos agrícolas en Cuba

Osmel Rodríguez González[1]✉, René Florido Bacallao[2] and Mario Varela Nualles[2]

ABSTRACT. A bibliographic review was carried out on the applications of modeling and simulation in agricultural crops with the aim of publicizing the characteristics and importance of their use, as tools for the estimation of crop yields. A summary of some concepts related to the modeling of agricultural crops and the main types of models that can be used from the point of view of their use in agriculture.

Key words: estimation, simulation models, yield

RESUMEN. Se realizó una revisión bibliográfica sobre las aplicaciones de la modelación y simulación en cultivos agrícolas en Cuba con el objetivo de dar a conocer las características e importancia del uso de los mismos, como herramientas para la estimación de los rendimientos de los cultivos agrícolas. Se presenta un resumen de algunos conceptos relacionados con la modelación de cultivos y los tipos principales de modelos que se pueden utilizar desde el punto de vista de su uso en la agricultura.

Palabras clave: estimación, modelos de simulación, rendimiento

INTRODUCTION

In the decades of the twentieth century and with force in the present, the techniques and technologies called Information Systems (IS) have had a vertiginous development, being used in industry, business administration and in many aspects of daily life. This development has been linked very directly to the improvement of computers and the possibilities of obtaining, ordering and effectively processing databases (1).

With the increasing capacity of computers and immense research in the field of computer science, new tools are provided to support the decision-making process in various disciplines and areas of design and management of the industry. Modeling and simulation are some of the most important and interdisciplinary tools (2).

The use of simulation models has been extended in agriculture and constitutes a tool for research, as well as for producers and technical advisors, who can now define, in the virtual universe of a computer, which is the best management practice for a crop in a certain season and productive site (3). The precise estimation of agricultural yields is of great theoretical and practical importance; but it is an issue not yet fully resolved (4), for performance is the final result of a group of interactions, where genotype, climate, soil and crop management intervene. Within the context of Cuban agriculture, crop modeling is a new discipline and literature on this subject is considered scarce (5).

That is why the objective of this work is to make known the characteristics and importance of the use of crop modeling and simulation, as a main tool in the decision-making processes, in order to apply these in the estimation of productive capacity in different climatic and management conditions in Cuba.

[1] Departamento Informática. Universidad Agraria de la Habana

[2] Instituto Nacional de Ciencias Agrícolas (INCA). Gaveta postal No.1, San José de las Lajas. Mayabeque, Cuba. CP 32700

✉ osmelr@unah.edu.cu

Definition of model

It can be said that a model is the ideal representation of a system and the way in which it operates, in which important features of the object under study are highlighted, whose objective is to analyze or predict future behavior (6,7).

It is par excellence the main tool that uses statistics to symbolize problems or situations in life. A particular case is the mathematical models.

Types of models

There are different classifications of the models. One of the most general is presented by Torres (8); according to which a model can be a replica of the object it represents. Changing the scale or the material with which it is constructed is called isomorphic and when it has some degree of abstraction of the object it represents, taking into account only the most relevant variables, it is called homomorphic.

It is important to point out that at a higher level of abstraction there is less degree of similarity and uniformity between the model and object represented and vice versa (9).

According to the level of abstraction, physical and scale models can be grouped as isomorphic and analog, mathematical and simulation models as homomorphic.

Mathematical models are the representation of a system by means of mathematical equations or statistical distributions of random values (10), allowing to obtain scientific knowledge whose veracity is verified in the course of social practice (11). The mathematical models that are most used in agronomy can be categorized into two classes: the only predictive or empirical models and the causal or mechanistic models (12).

The first ones are descriptive; they are derived from observed data without involving physiological processes and have little explanatory capacity (13). They are generally expressed as regression equations (with one or several factors) and are used to estimate the final production. Examples of such models include the response of production to the application of fertilizers, the relationship between the area of the leaf and the number of leaves of a given plant, the relationship between the height of the stem and the number of stems, its diameter and the final production.

On the contrary, the mechanistic models have an explanatory capacity of the physiology of the crop, because they consider aspects such as temperature, photosynthetically active radiation, leaf area index, photosynthesis, respiration and efficiency in the use of radiation (13). These models have the ability to imitate important physical, chemical or biological processes and describe how and why a particular response results. The analyst usually starts with some empiricism and as knowledge is gained, variables and additional parameters are introduced to explain the production of the harvest. Thus, the analyst adopts a reductionist approach. Most crop growth models fall into this category.

Mathematical models of crop growth have been developed to assess the impact of climate change on food production and to develop strategies such as risk management of the application of pesticides and fertilizers in the agricultural setting (14); so that the available resources can be rationalized.

For the processing and analysis of the problem, using regression analysis, it is necessary to consider (15):

- Plotting points to analyze data trend.
- Selection of the type of model to be adjusted
- Setting the model, with the support of appropriate software
- Description of the process from the obtained model

These studies require an efficient work in the organization and development of scientific research and the knowledge it generates, which can greatly contribute to the consistent application of these models, with the support of new information technologies and the communication. Also, very often, the theoretical assumptions of the statistical models are not assessed and no valid conclusions are drawn from the information analyzed (16).

For the selection of the model that best fits the data of the experiment, fundamentally in the explanatory and dynamic models, the following criteria must be taken into account (16):

- Methods of adjusting the models
- Standard error of the estimators of the parameters (Student's t test)
- Coefficient of variation of the estimators
- Confidence limits of the parameters.
- Test of redundancy of the parameters
- Analysis of variance related to the model in question.
- Coefficient of determination R_2 and R_2 adjusted by the degrees of freedom, for models with different numbers of parameters
- Sum of squares or Residual Half Square
- Standard estimation error
- Test of lack of adjustment of the model
- Analysis of the effect of the use of transformations in the model
- Diagnosis and treatment of multicollinearity, in multiple linear regression models

- Validation of model predictions
- Statistical PRESS (Sum of Squares of Prediction Error)
- CMEP Statistics (Average Square Prediction Error)
- Statistic Cp de Mallows
- Correlation coefficients between predicted and actual results
- Analysis of the accuracy of the estimates
- Analysis of waste
- Normality (Test of Shapiro-Wilks, Kolmogorov-Smirnov)
- Autocorrelation (Streaks Test, Signs, Durbin-Watson, X2 Independence, Ljung and Box)
- Homocedasticity (Residue graphs, Cochran, Bartlett and Hartley test)

Several authors suggest that a crop model is the dynamic simulation of crop growth by the numerical integration use of the constituent processes with the help of computers (17,18). But although in many works modeling and simulation are usually seen as a single process, they must be differentiated (9), since modeling can be done without the need for simulation, not so for the latter, which cannot be done but exists the modeling process.

Crop simulation models

The simulation can be defined as a numerical technique for the realization of experiments with certain types of mathematical models, which describe the behavior of a complex dynamic system, through the use of computation (19). It is the process of putting the model to work to see if it is correct or not.

The concept of simulation must be dynamic mainly for two reasons: it is the best understanding of the processes involved in the production of crops as well as the resolution of problems. Models that serve as a tool to solve models are usually multidisciplinary models (20).

Crop growth simulation models and the analysis of the soil-plant-atmosphere system are important tools for modern agricultural research (21). A crop model represents in a simple and synthetic way the most important physiological and ecological processes that govern growth, using mathematical equations (20). The understanding of the functioning and evolution of the main factors responsible for these conditions is acquired by comparing simulation results with experimental observations. These observations can be designed to validate the model, taking into account the weather, soil conditions and crop management, according to the place of implantation (22). Once this first validation stage of the model has been carried out, it can be used to help analyze and interpret different future scenarios due to modifications that may be proposed in the management of the crop, changes in climatic conditions or for the forecast of yield, among others indicators.

These models can increase the understanding and management of the agricultural system in a holistic way. Crop simulation models have been used to investigate the behavior of different cultivars in a range of sowing dates in relation to different soil-climate scenarios.

Simulation models are presented as an alternative to be used in future scenarios (23). Although the simulation technique is generally seen as a method of last resort, the advances in simulation methodologies and the availability of software that currently exists in the market have made it possible for the simulation technique to be one of the most used tools in the field system analysis, because it has advantages such as (24):

- Allows studying real systems that cannot be evaluated analytically.
- It makes it possible to estimate the behavior of an existing system, if some of the current operating conditions are modified.
- It can compare different design alternatives (or ways of operating a system), before building it, to see which one behaves better.
- It allows studying in a short time the evolution of a system in a long period of time (years of experience in the real system can be evaluated in a few minutes of simulation).

Numerous models have been developed by different work groups and each of them has strengths and weaknesses to predict the response variables (25). So it is very important, before adopting one or another model for agricultural and environmental applications that an evaluation and exhaustive validation work is carried out (26).

Applications of mathematical models and the simulation of agricultural crops in Cuba

Since the 1980s modeling have been used by some Cuban researchers with the aim of describing the growth of cultivars. In the National Institute of Agricultural Sciences (INCA), comparisons were made between several mathematical functions to describe the growth of some organs in coffee postures grown in the nursery (27). The influence of the use or not of the shade during the nursery period, the height above sea level and two sowing dates, in the growth dynamics of coffee plantlets was also determined (28).

Models have also been compared to measure the response to nitrogen doses in corn and coffee. This showed that a discontinuous rectilinear model is more suitable in the recommendation system of optimal doses of nitrogen fertilizers for these crops, in comparison to the curvilinear model, which provides superior optimal doses, which imply a lower partial factor of productivity of the recommended dose. (29)

Other works applied modeling tools for the analysis of responses of plant-environment-management interactions in different scenarios of rice, corn, sorghum and wheat production in Cuba (30); demonstrating that these allow establishing strategies for the development of crops studied in future scenarios and in other growing conditions. For the first time in the country, information is available to predict the performance and performance of these cereals. They also show that the DSSAT model can be used for the conditions of Cuba in the performance indicators and their components.

For the benefits of deterministic models, specialists from the Agrarian University of Havana and the National Institute of Irrigation and Drainage, develop the first works to use, in the geographical conditions of Cuba, five of the international agrohydrological models. For the first time in Cuba, the parameters describing the hydraulic properties for the main groups of Cuban soil are given. This research gives validity to the SWACROP model to be used in the cultivation of potatoes in tropical conditions (31).

Some works have been carried out, during several years, in the evaluation and validation of different models of simulation of water transfers for the edaphoclimatic conditions of the south region of Havana (now Mayabeque and Artemisa). These investigations aim to summarize and discuss the main characteristics of the STIC and MACRO simulation models and their possibilities for the prediction of the behavior of agricultural crops in the face of different water management, fertilization and climatic environments (32-35).

Mathematical modeling has also been used in order to study the growth of the average diameter, average height and volume per hectare of *Pinus caribaea* morelet var. *Caribaea barret* and golfari in the Silvicultural Unit "Los Jazmines" of Viñales and in the Integral Forestry Company "La Palma" (36-38) where the models that best describe the behavior of the variables studied in each of the scenarios were determined.

The application to the production of pastures and forages had its beginnings in star grass (*C. nlemfuensis*) (39-41), where polynomial relationships between the variables were found; but in these models the parameters do not have biological interpretation. Subsequently, the productive behavior of the star grass (*C. nlemfuensis*) was modeled and simulated under different cutting frequencies, fertilization levels and adverse climatological conditions for the development of this crop (42). The Gompertz model used adjusted the data with coefficients of determination that were around 99 % for the rainy and dry periods. It also predicts that an increase in the average temperature of the planet by 2 and 4 ºC will affect the accumulated dry matter yield of this crop mainly in the dry season.

Other authors studied the biomass accumulation dynamics of king grass (*P. purpureum*) or some of its clones with the use of non-linear models (43-47). In addition, in cultivars of *Brachiaria*, *Panicum* and *Pennisetum* and in varieties of *Tithonia diversifolia*, different models were evaluated and found linear adjustments for the variables under study (48,49).

CONCLUSIONS

The advancement of information systems in agriculture, together with the development of computing has allowed an accelerated development of agricultural crop modeling. These are a useful tool to predict the behavior of crops under specific conditions; integrating knowledge of physiology, soil science and meteorological data. They also let the development of an efficient agriculture.

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